Trade Policy in a Sector-Focused General Equilibrium Model

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August, 1998 (revised)

Abstract: In this note we extend a single sector model to include general equilibrium effects, developing a simple, sector-focused applied general equilibrium model. The model includes Armington preferences for imports (i.e. they are imperfect substitutes) in the focus sector. Moving from partial to general equilibrium, the single market assumption is dropped and a reduced form residual sector is explicitly added. This allows us to highlight differences between partial equilibrium and simple general equilibrium Walrasian closure rules, and to examine partial equilibrium measures of the effects of protection in a general equilibrium context.

Introduction

This note builds on Francois and Hall (1997) to develop a simple, sector-focused applied general equilibrium model. The model involves an extension of a standard partial equilibrium Armington model, where the single market assumption is dropped and a reduced form residual sector is explicitly added. This allows us to highlight differences between partial equilibrium and simple general equilibrium closures, and to examine partial equilibrium measures of the effects of protection in a general equilibrium context.

The Model

The market for the focus sector good

Following Francois and Hall (1997), we first specify Armington-type preferences as a system of non-linear equations. We define the Armington composite good \( q \) as a CES composite of the domestic good \( X_1 \), and of imports \( X_i \) from countries \( i=2..n \).

\[
q = \left( \sum_{i=1}^{n} \alpha_i X_i^\rho \right)^{1/\rho}, \text{ where } \rho > 1
\]
In calibrating the model, we scale quantities so that internal prices are all unity in the benchmark. This includes the price for the Armington composite good $q$. The price index for the composite good can be shown to equal:

$$\begin{align*}
(2) \quad P &= \left[ \sum_{i=1}^{n} \alpha_i^\alpha P_i^{1-\alpha} \right]^{1-1/\rho} \\

\end{align*}$$

At the same time, from the first order conditions, the demand for good $X_i$ can be shown to equal

$$\begin{align*}
(3) \quad X_i &= \left[ \alpha_i/P_i \right]^\alpha \left[ \sum_{i=1}^{n} \alpha_i P_i^{1-\alpha} \right]^{-1} E \\
&= \left[ \alpha_i/P_i \right]^\alpha P_i^{\sigma-1} E \quad \text{where} \quad \rho = 1 - (1/\alpha)
\end{align*}$$

where $E$ is expenditure on the composite good $q$.

Combined with supply equations, the terms defined in equations (3) can be used to define a simple non-linear system for the focus market in terms of prices. In particular, if we specify supply as a function with constant supply elasticity $\epsilon_{si}$, then excess demand (i.e. market clearing) conditions in each market are defined as follows.

$$\begin{align*}
(4) \quad \left[ \alpha_i/P_i \right]^\alpha P_i^{\sigma-1} E - K_{si} P_i^{\epsilon_{si}} &= 0 \\

\end{align*}$$

At the same time, the composite price equation can be re-written as follows.

$$\begin{align*}
(5) \quad \left[ \sum_{i=1}^{n} \alpha_i^\alpha P_i^{1-\alpha} \right]^{1-1/\rho} - P &= 0
\end{align*}$$
In a partial equilibrium model, we can close this system of equations by defining demand for the composite good as

\[ q = k_A P^{\text{NA}} \]

where NA is the elasticity of demand for the composite good. This allows us to specify excess demand for the composite good as follows. Note that, in equations (3) and (4), \( E = Pq \).

\[ k_A P^{\text{NA}+1} - E = 0 \]

Equations (4), (5), and (7) define a system of \((n+2)\) equations and \((n+2)\) unknowns. The system can be solved for prices, and solution prices then used to solve for quantities and welfare measures.

Tariffs or other price-based measures of import protection can be added to the system through the import supply functions. This requires a slight modification of equation (4), as follows.

\[ \left[\frac{\sigma_j}{P_j}\right]^0 P^{\sigma_j - 1} E - K_{si} \left( P_j/(1 + t_j)\right)^{\sigma_j} = 0 \quad \text{for } j = 2..n \]

### Moving to General Equilibrium

To relax the partial equilibrium assumption and explicitly introduce the rest of the economy, we specify a constant elasticity of transformation (CET) function between production of \(X_1\) and a composite good \(G\), which represents the composite value of activity in the rest of the economy. Formally, we define the CET function:

\[ \overline{\xi} = \gamma X_1^\zeta + (1 - \gamma) G^{\zeta/(\zeta-1)} \], \( \zeta > 1 \)

We assume that the numerarie is \(G\). Differentiating equation (9), we can show that, around the region of the benchmark equilibrium, the elasticity of transformation, \(1/(\zeta - 1)\), approximates the price elasticity of supply for \(X_1\). This means that we can calibrate the curvature of the CET from the estimated supply elasticity for the \(X\) sector.

\[ \epsilon_{s1} = 1/(\zeta - 1) \]
In a free-trade equilibrium, the internal price line for good $X_1$, $-P_1$, will be tangent to the transformation locus, with the slope of the transformation locus defined by the condition:

$$(11) \quad P_1 = \left( \frac{\gamma}{1-\gamma} \right) \left( \frac{X_1}{G} \right)^{\zeta^{-1}}$$

The combination of the composite good activity and the transformation technology between $X_i$ and $G$ also implies a concave transformation locus defined over $G$ and the CES composite $q$. Again, in a free-trade equilibrium, for a small country we expect a tangency between the internal price line $-P$ and the slope of the $G-q$ transformation locus. For a country that is large in the market for $X$, potential terms-of-trade effects imply a non-tangency between prices and the $G-q$ transformation locus.

For parameterization of the CET function, the terms $\gamma$ and $z$ can be determined from benchmark data. In particular, if we normalize benchmark domestic prices to unity, then

$$(12) \quad \gamma = G^{\zeta^{-1}}/(X_1^{\zeta^{-1}} + G^{\zeta^{-1}})$$

and $z$ can be calculated from the set of benchmark parameter and variable values.

To represent the demand-side of the system, we adopt Cobb-Douglas preferences defined over $q$ and $G$, replacing equation (7) with

$$(7)' \quad \theta_{\text{G}} + \theta_{\text{P1}}B_{\text{P1}}^{\zeta_{-1}} + T = E = 0$$

where $\theta$ is the Cobb-Douglas expenditure share for $q$ (derived from benchmark data), $T$ is tariff revenue, and all other variables are as defined above.

Finally, to close the general equilibrium system, we need to add an equation for the determination of $G$. This can be done by rearranging equation (9), and substituting the supply equation for $X_1$.

$$(12) \quad z = \left[ \gamma \theta_{\text{P1}} P_1^{\zeta_{-1}} + (1-\gamma) G^{\zeta} \right]^{\zeta_{-1}} = 0$$

Once we substitute the price-based definition of $T$ into equation (7)', equations (4), (5), (7)', and (12) define a system of (n+3) equations and (n+3) unknowns: $P_{x,E,P}$, and $G$. 

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The system can be solved for prices, and solution prices then used to solve for quantities. Welfare changes can then be measured directly as equivalent variation:

\[ EV = \frac{(Y_1 / P_1)^\theta G_1^{1-\theta}}{(E_0 / P_0)^\theta G_0^{1-\theta}} - 1 \]  

*Implementation*

Table 1 presents a simple spreadsheet implementation of the basic model described above, with imports differentiated by origin from two sources.

*References*
